TU Clausthal

# Benchmarks on Boolean Function Synthesis / Quantifier Elimination for Reactive Synthesis

**Dagstuhl Seminar 24171**

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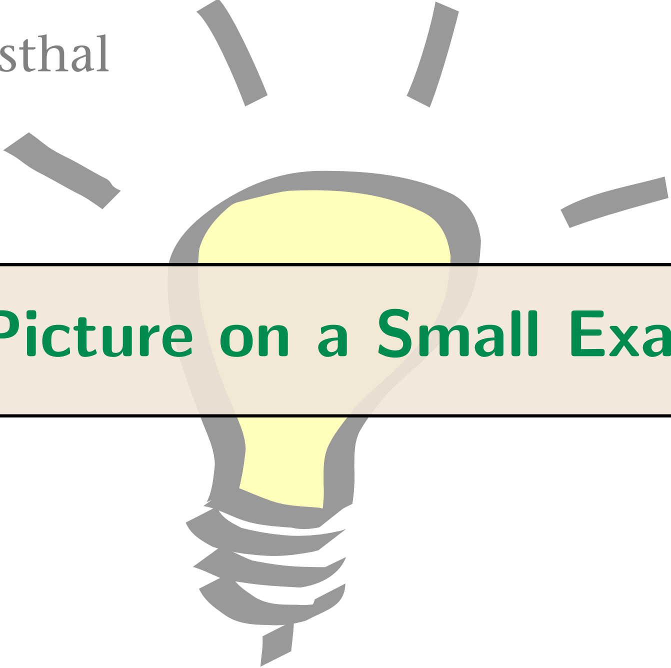
April 2024



# Big Picture on a Small Example

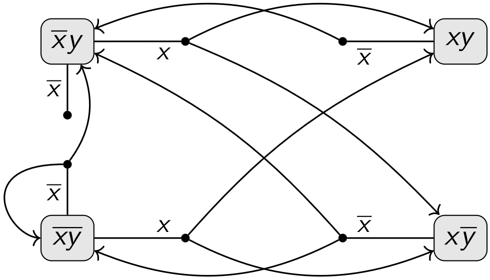
# Example for Using Fixpoint Formulas: GR(1) Synthesis

Game:

$\mathbf{G}(\neg x \vee \neg \mathbf{X} x)$
$\wedge\ \mathbf{G}((\neg x \wedge y) \rightarrow \mathbf{X} x)$
$\wedge\ \mathbf{G}((\neg x \vee \neg y \vee \mathbf{X} x \vee \mathbf{X} \neg y)$

Specification:

$\ldots$
$\rightarrow \mathbf{GF}(x \wedge y) \wedge \ldots$




$$\phi(x, y, x', y') = (\neg x \vee \neg x') \wedge (x \vee \neg y \vee x') \wedge (\neg x \vee \neg y \vee x' \vee \neg y')$$

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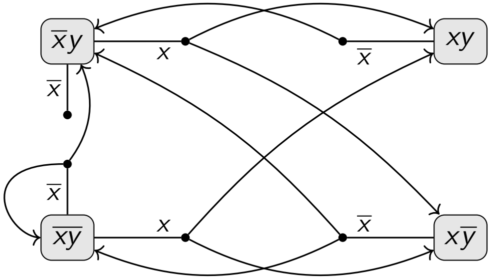


$$\nu X.\mathrm{CPre}_0((\psi(x,y) \vee X))$$

$\psi(x,y)$ is just something to abstract that we normally have a more complicated fixpoint formula, in the following it is **true**

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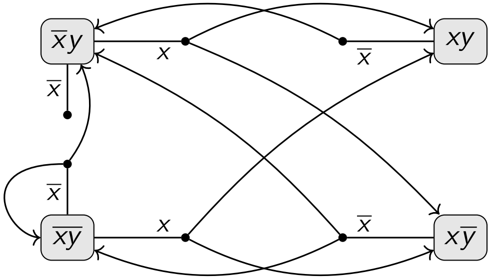


$$\nu X.\mathrm{CPre}_0((\psi(x,y) \vee X))$$
$$= \nu X.(\ \exists x, y. \forall x'. \exists y'. \phi(x,y,x',y') \wedge (\psi(x',y') \vee X)\ )$$

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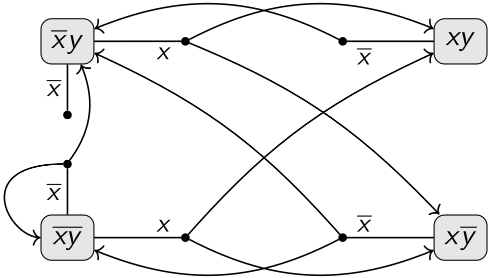


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Step 1: $X = \mathbf{true}$

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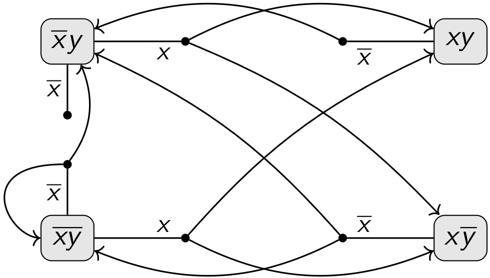


$$\nu X.\mathrm{CPre}_0((\psi(x,y) \vee X))$$

$$= \nu X.(\ \exists x, y.\forall x'.\exists y'.\phi(x,y,x',y') \wedge (\psi(x',y') \vee X)\ )$$

Step 1: $X = \mathbf{true}$ Step 2: $X = (x \vee \neg y)$

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Specification:

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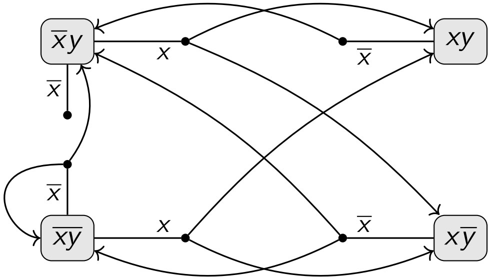


$$\nu X.\mathrm{CPre}_0((\psi(x,y) \vee X))$$
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Step 1: $X = \mathbf{true}$ Step 2: $X = (x \vee \neg y)$

Step 3: $X = (x \vee \neg y) \wedge (\neg x \vee \neg y)$

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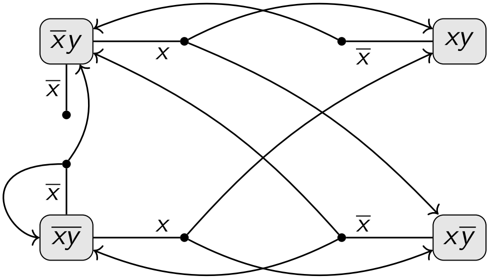


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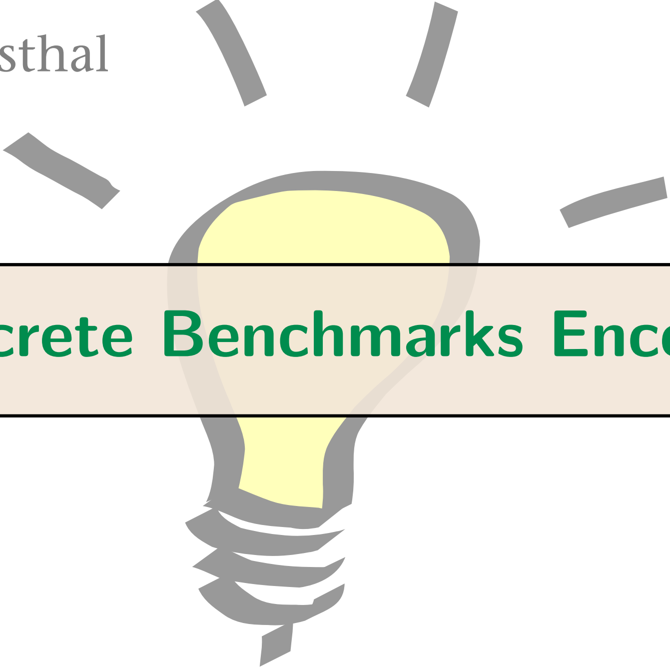
Step 1: $X = \mathbf{true}$ Step 2: $X = (x \vee \neg y)$

Step 3: $X = (x \vee \neg y) \wedge (\neg x \vee \neg y)$ Step 4: same as step 3



# Concrete Benchmarks Encoded

# Encoded Problem

## Encoded problem

$$\nu A \dots \mu B \dots \nu C.\quad \underbrace{\Finv X, Y.\forall X'.\exists Y'.\neg \overbrace{\varphi^A(X, Y, X')}^{\text{Currently enc'd as BDD}} \overset{\text{Encoded using helper variable}}{\vee} (\overbrace{\varphi(X, Y, X', Y')}^{\text{Naturally in CNF}} \wedge \overbrace{\psi(A, B, C))}^{\text{Currently enc'd as BDD}}}_{\text{Problem encoded as Free QBF instance}}$$

## Notes

- $X$, $Y$, $X'$, $Y'$ are variable sets, $\Finv$ stands for free variables
- $\psi(A, B, C)$ will is computed from the previous result of solving this problem and ranges over the variables in $X'$ and $Y'$
- The BDDs are encoded using innermost existentially quantified helper variables – one for each node.

# References I

Roderick Bloem, Barbara Jobstmann, Nir Piterman, Amir Pnueli, and Yaniv Sa'ar. Synthesis of reactive(1) designs. *J. Comput. Syst. Sci.*, 78(3): 911–938, 2012. doi: 10.1016/J.JCSS.2011.08.007. URL https://doi.org/10.1016/j.jcss.2011.08.007.